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Controlling spatiotemporal chaos using multiple delays

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A control method for manipulating spatiotemporal chaos is presented using lumped local feedback with several different delay times. As illustrated with the two-dimensional Ginzburg-Landau and the Fitzhugh-Nagumo equation this method can, for example, be used to convert chaotic spiral waves into guided plane waves and for trapping spiral waves.

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Many spatially extended, nonlinear systems exhibit spatiotemporal chaos in terms of irregular wave fronts or turbulent spiral dynamics [1]. This kind of complex dynamics occurs, for example, with catalytic carbon monoxide oxidation on a platinum (110) surface [2–4], liquid crystals [5], cardiac tissue [6], or electrochemical reaction diffusion systems [7].

Often, however, spatiotemporal chaos is not wanted. The healthy human heart, for example, generates plane waves traveling around the heart muscle. As a result of irregularities or disease, these plane waves can split into several spiral waves leading to severe arrythmias like irregular oscillations or fibrillation. In technology, electrocatalysis in fuel cells, corrosion, electrochemical machining of metals, or the generation of pattern and clusters are often governed by complex spatiotemporal dynamics. All these examples have in common, that strategies are required to manipulate and control the system of interest. Therefore, in the past 10 years different approaches have been devised for taming spatiotemporal chaos [8]. Since we are dealing with spatially extended systems the influence of boundary conditions or small spatial inhomogeneities in the medium can be exploited to control the dynamics. Such static methods were used to generate drifting spiral waves [9] and to suppress chaotic spiral dynamics [10].

An active manipulation of the dynamics can be implemented by external periodic forcing [11–13] or short pulses such as electrical shocks used to eliminate spiral waves in cardiac tissue to reset the heart muscle contractions [14]. Furthermore, feedback control is used in various forms. With proportional control one or several suitably chosen observables of the considered system are used to generate the feedback signal, which is then amplified by some gain factor and fed back to the system. For spatially extended systems this can be done locally or globally using a mean field signal, for example, to control spreading of microscale liquid films [15].

Chaos control using delayed feedback based on the amplified difference of a measured signal and its delayed component was first proposed by Pyragas [16] and turned out to be very efficient for experimental applications. This approach is also called time delay auto synchronization (TDAS) and it is mainly used for stabilizing unstable periodic orbits embedded in some chaotic attractor. For many examples it turned out that the performance of this control method can be improved by including integer multiples of the (fundamental) delay time in the feedback signal [17] [so-called extended TDAS (ETDAS)].

Time delayed feedback was also applied to control spatially extended systems including the one-dimensional chaotic Ginzburg-Landau equation [18,19], spatiotemporally chaotic semiconductor laser arrays [20], spiral waves in spherical surfaces [21], and stabilization of rigid rotation of spiral waves in excitable media [22]. Furthermore, the efficiency of the delayed feedback methods was significantly improved by spatially filtering the applied control signal [23,24].

All of the above-mentioned control methods are based on a single delay time and symmetric gain factors. Recently, however, it has been shown that stabilization of steady states (fixed points) [25] can significantly be improved by using not only a single delay time (and its integer multiples [17]) but two or more independent delay times with asymmetric gains [26]. This multiple delay feedback control (MDFC) was successfully applied to stabilize different chaotic systems [27,30]. In particular, it turned out to be very efficient to suppress irregular intensity fluctuations of intracavity frequency doubled solid state lasers [26] where chaos limits technical applications (e.g., holographic displays) requiring constant light output.

Here, we shall demonstrate how MDFC can also be used to (locally) stabilize and manipulate spatiotemporal chaos. As our first example we employ the two-dimensional complex Ginzburg-Landau equation (GLE)

$$\partial_t f = (1+ia)\nabla^2 f + f - (1+ib)f|f|^2 + u \tag{1}$$

with an external control signal $u(\mathbf{x},t)$. ∂_t and ∇ denote the temporal and the spatial derivative, respectively. The GLE (1) possesses an unstable steady state solution $f(\mathbf{x},t)=0$ which can be stabilized by means of a P controller or MDFC. Furthermore, harmonic waves

$$f(\mathbf{x},t) = f_0 e^{i(\mathbf{k_0}\cdot\mathbf{x}-\omega_0 t)} \tag{2}$$

with wave vector $\mathbf{k}_0$, frequency ω_0 and amplitude f_0 comprise unstable solutions of the GLE [28]. Substituting (2) into the GLE (1) one obtains the relations $\omega_0=k_0^2(a-b)+b$ and $f_0=\sqrt{1-k_0^2}$ where $k_0^2=\|\mathbf{k}_0\|^2\leqslant 1$. In the one-dimensional case this kind of unstable periodic orbits (UPOs) embedded in the chaotic attractor of the system can be stabilized by means of TDAS [18,19]. For higher dimensional systems, however, it was shown in Ref. [28] that for $ab<-1$ perturbations exist that cannot be controlled by (E)TDAS [29].

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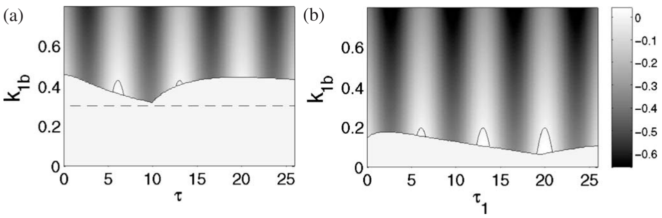

FIG. 1. Stabilization of plane wave solutions (2) of the GLE (1) with $a=1.1$ and $b=-1$ using the delayed feedback signal (3). In the gray shaded regions of the control parameter space τ_1-k_{1b} stabilization is successful and results in the gray scaled wave number shift Δk^2 (5) vanishing for parameter combinations indicated by the ∩-shaped curves. (a) Single delay feedback with $k_{1a}=0.3$. The dashed line denotes the case of symmetric feedback ($k_{1a}=k_{1b}$) that fails to stabilize plane waves. (b) MDFC with two delay times and fixed control parameters $k_{1a}=0.3, k_{2a}=0.1, k_{2b}=0.3, \tau_2=7.2$.

We shall show now that this limit can be overcome by using asymmetric delayed feedback and that the stability range can be extended by using several independent delay times. To stabilize plane waves (2) we use the MDFC signal

$$u(\mathbf{x},t) = \sum_{m=1}^{M} k_{ma} f(\mathbf{x},t-\tau_m) - k_{mb} f(\mathbf{x},t) \tag{3}$$

with gains k_{ma}, k_{mb} and delay times τ_m. Furthermore, we assume that the controlled plane wave is given as $f(\mathbf{x},t) = f_c e^{i(\mathbf{k_c}\cdot\mathbf{x}-\omega_0 t)}$. If this plane wave is substituted in (3) one obtains $u(\mathbf{x},t)=T(\omega_0)f(\mathbf{x},t)$ with transfer function $T(\omega_0) = \sum_{m=1}^{M} k_{ma} e^{-i\omega_0\tau_m} - k_{mb}$. Inserting the plane wave solution and the corresponding control term into the GLE (1) results in

$$\begin{aligned} 1 - k_c^2 - f_c^2 + \mathrm{Re}[T(\omega_0)] &= 0, \\ \omega_0 - a k_c^2 - b f_c^2 + \mathrm{Im}[T(\omega_0)] &= 0, \end{aligned} \tag{4}$$

where $k_c^2 = \|\mathbf{k_c}\|^2$. Combining both constraints of Eq. (4) we obtain the dispersion relation $k_c^2 = k_0^2 + \Delta k^2$, where

$$\Delta k^2 = \frac{b\,\mathrm{Re}[T(\omega_0)] - \mathrm{Im}[T(\omega_0)]}{b-a} \tag{5}$$

describes the wave number shift due to the feedback control. Since $k_0^2 \leqslant 1$ the relation $k_c^2 \geqslant 0$ is fulfilled if $\Delta k^2 \geqslant -1$. If the condition $b\,\mathrm{Re}[T(\omega_0)] = \mathrm{Im}[T(\omega_0)]$ holds, Δk^2 vanishes, and control results in a plane wave with the same wave number $k_c = k_0$ as for the free running system but different amplitude $f_c = \sqrt{f_0^2 + \mathrm{Re}[T(\omega_0)]}$.

The magnitude of the wave number shift depends on the transfer function $T(\omega)$ that can be adjusted with the parameters of the control signal. To illustrate this dependence we show in Figs. 1(a) and 1(b) the value of Δk^2 (gray scaled) in the control parameter plane τ_1-k_{1b} for MDFC with one and two delay times, respectively. Below some critical values of the gain k_{1b} control fails and the plane wave remains unstable (light gray shading in Fig. 1). Since the parameter values of the GLE are in this case $a=1.1$ and $b=-1$ the TDAS controllability criterion $ab > -1$ derived in Ref. [28] is not fulfilled. Therefore, symmetric delayed feedback control with $k_{1a}=k_{1b}$ fails as can also be seen in Fig. 1(a) where the dashed line at $k_{1b}=0.3=k_{1a}$ lies in the unstable region. In contrast, asymmetric delayed feedback enables stabilization if the gain k_{1b} is sufficiently high, including parameter combinations (τ_1, k_{1b}) where Δk^2 vanishes.

Similar to the results obtained with several other dynamical systems [26,27] application of an additional feedback loop with a different delay time $\tau_2=7.2$ results in increasing stability, here visible as a reduced size of the unstable region shown in Fig. 1(b).

Feedback control that is homogeneously applied in space [like that in Eq. (1)] is in general difficult to implement experimentally. Any experimental sensor of finite size will measure the activity of the process of interest in terms of spatial averages in some sensor region and any control signal is practically applied not to points but to extended and lumped actuator areas. Therefore, we shall try now to stabilize plane waves with a small number of control cells describing small spatial areas where spatially averaged observations are measured and/or where the control signal is applied. The corresponding GLE (1) is solved numerically (for periodic boundary conditions) with a spectral code based on a Runge-Kutta scheme of fourth order combined with a spectral method in space with a spatial grid of 90×90 elements ($\Delta x = \Delta y = 1$).

As our first example of lumped MDFC we want to show how turbulent dynamics [Fig. 2(a)] can locally be turned into plane waves. The control cells are located on two parallel lines as indicated by white rectangles in Fig. 2(b). For each control cell $\mathcal{C}_j$ average values $f_j(t)$ of the complex amplitude are computed to simulate local sensors. These average values enter the local control signal

$$\begin{aligned} u_j(t) = {} & k_{1a} f_j(t-\tau_1) + k_{2a} f_j(t-\tau_2) + k_{3a} f_j(t-\tau_3) \\ & - (k_{1b} + k_{2b} + k_{3b}) f_j(t), \end{aligned} \tag{6}$$

which is within the control cell $\mathcal{C}_j$ added to the GLE (1). The definition of the local control signal is also illustrated in Fig. 2(e). This local control scheme succeeds in generating plane waves, although for the GLE parameters $(a,b)=(1.1,-1)$ used, the TDAS-controllability condition derived for spatially homogeneous feedback [28] is not fulfilled.

In Figs. 2(c) and 2(d) it is shown how chaotic spiral waves can be converted to traveling plane waves with constant velocity. To visualize the temporal dynamics, Fig. 2(f) shows the phase values in a vertical section of the x-y plane at $x=-10$ as a function of time t. With MDFC switched on at $t=300$ spiral waves occurring between the control cells are converted into plane wave fronts that are accelerated until they reach their (constant) maximum speed for $t > 600$. The velocity of these waves can be adjusted by varying the delay times, the gain factors, or the vertical distance of control cells. To demonstrate this feature of MDFC the control parameters are switched to new values at $t=800$. Now plane waves with larger wavelength and smaller velocity are stabilized by the control scheme. This change of the temporal structure is visualized in Fig. 2(g) where the real part of the solution f at some fixed point in the controlled area is plotted versus time. As can be seen the period of the local oscillations associated with the traveling wave can be varied by a

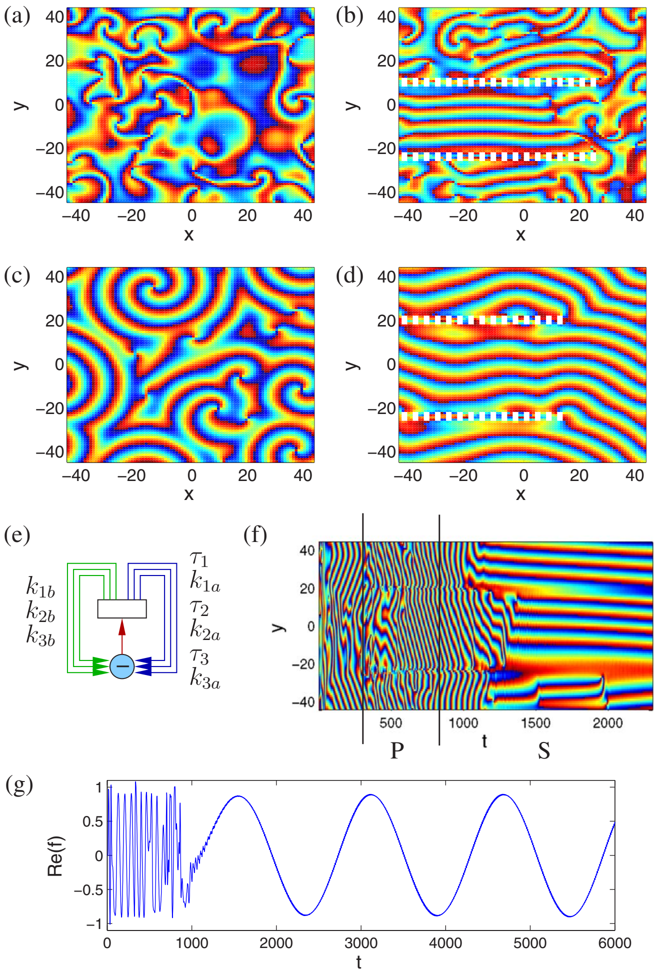

FIG. 2. (Color online) Phase of the complex solution f of the two-dimensional GLE (1). Without feedback chaotic turbulent dynamics (a) and spiral waves (c) occur. With MDFC Eq. (6) applied at some control cells (marked in white) both types of dynamics can be converted to plane waves (b),(d) traveling with constant velocity (asymptotic dynamics). Here, signals from each control cell are fed back with and without delay as illustrated in (e) for a single control cell. (a,b) GLE parameters, $(a,b)=(1.1,-1)$ (controllability condition $ab>-1$ from Ref. [28] not fulfilled); MDFC parameters, $k_{1a}=0.33$, $k_{1b}=0.67$, $k_{2a}=0.365$, $k_{2b}=0.68$, $k_{3a}=0.405$, $k_{3b}=0$, $\tau_1=25$, $\tau_2=61.5$, $\tau_3=94$. (c)–(f) GLE parameters, $(a,b)=(-1.45,0.34)$ (controllability condition $ab>-1$ from Ref. [28] fulfilled); MDFC switched on at $t=300$ with parameter set $P=\{k_{1a}=0.13,k_{1b}=0.43,k_{2a}=0.4,k_{2b}=0.49,k_{3a}=0,k_{3b}=0,\tau_1=20,\tau_2=59,\tau_3=104\}$ with effectively two delay times [set P is also used in (d)]. At $t=800$ MDFC switched to control parameter set $S=\{k_{1a}=0.24,k_{1b}=0.42,k_{2a}=0.32,k_{2b}=0.21,k_{3a}=0.1,k_{3b}=0,\tau_1=20,\tau_2=59,\tau_3=104\}$ where the third delay loop is activated. As a result temporal oscillations at fixed locations drastically change their periodicity as shown in (f) for a vertical section of (d) at $x=-10$ and in (g) for Re(f) at the point $(-35,9)$.

factor of 10, an option that is important for many practical applications (e.g., controlling ventricular fibrillations).

The range of influence of the control cells (size 2×4) is given by the spatial correlation length Cor≈ 8 of the free running GLE (1) that is defined via the first maximum of the spatial correlation function. Numerical simulations indicate that stabilization is optimal if the mutual (horizontal) distances of the control cells equal about one-half of the correlation length. Gains are adjusted experimentally in combination with delay times near some roots and maxima of the temporal autocorrelation function. If only very few and therefore spatially widely separated control cells are used a phenomenon similar to diffraction occurs where spiral waves wriggle around the control cells without being influenced substantially.

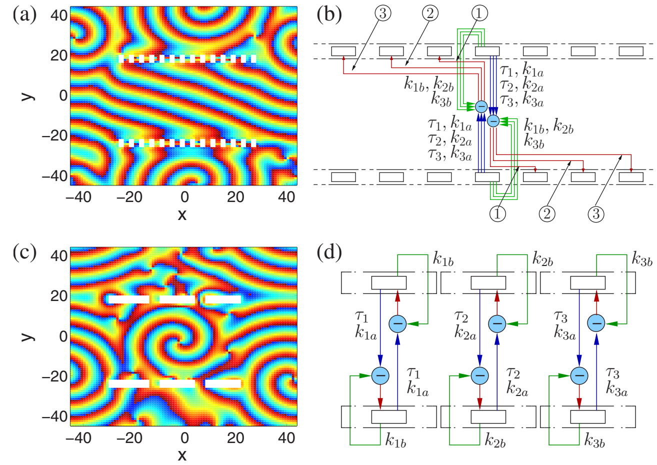

FIG. 3. (Color online) Asymptotic phase dynamics of the GLE (1) with $(a,b)=(-1.45,0.34)$. In the controlled region chaotic spiral waves are turned into slanted traveling waves (a) if the control scheme (b) is applied with parameters $\tau_1=31$, $\tau_2=59$, $\tau_3=84$, $k_{1a}=0.22$, $k_{1b}=0.3$, $k_{2a}=0.2$, $k_{2b}=0.5$, $k_{3a}=0.3$, and $k_{3b}=0$. Using control scheme (d) with $k_{1a}=0.22$, $k_{2a}=0.1$, $k_{3a}=0.35$, $k_{1b}=0.3$, $k_{2b}=0.5$, $k_{3b}=0.0$, $\tau_1=41$, $\tau_2=27$, and $\tau_3=49$ individual spiral waves can be trapped (c).

For the previous examples we used individual MDFC at each cell, i.e., signals from a given cell $\mathcal{C}_j$ were used to control the same cell [see Eq. (6) and Fig. 2(e)]. In general, (delayed) signals from different cells can be combined and the resulting control signal can be applied to some other cell. Examples for such a more sophisticated MDFC are presented in Fig. 3. The control cells are grouped in small line segments and their wiring (including three delays τ_1, τ_2, and τ_3) is given in Figs. 3(b) and 3(d). With the control schemes shown it is possible to stabilize slanted plane waves running with a (variable) angle between the two rows of control elements [Fig. 3(a)] or to trap spiral waves in the controlled region [Fig. 3(c)] where the rotation direction of the spiral wave can be manipulated by changing the feedback parameters.

As a second example of a spatially extended system with complex dynamics we use the two-dimensional Fitzhugh-Nagumo equation (FNE)

$$\frac{\partial u}{\partial t} = \nabla^2 u + \epsilon^{-1} u(1-u)\left(u - \frac{v+b}{a}\right),$$

$$\frac{\partial v}{\partial t} = D_v \nabla^2 v + u - v, \qquad (7)$$

that is solved numerically for periodic boundary conditions using the code of Barkley [31] (Euler integration in time and

9-point laplacian on a 90×90 grid in space). For $a=0.75$, $b=0.06$, $\epsilon=1/12$, and $D_v=0$ spatiotemporal chaos occurs as shown in Fig. 4(a). To turn this turbulence into running waves a row of control cells is introduced where spatially averaged signals of the variable u are measured and then fed back with different time delays τ_m. At each control cell $\mathcal{C}_j$ the feedback signal is added to the u equation in (7) to turn the turbulent dynamics into traveling waves emanating from the row of control cells [Fig. 4(b)].

These examples indicate the potential of multiple delayed feedback applied to spatiotemporal systems providing interesting options for specific manipulations of complex spatiotemporal structures, such as guided plane waves, or trapped spiral waves.

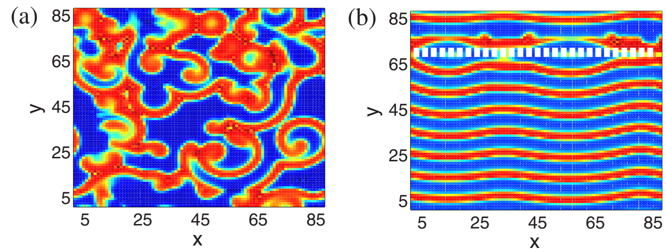

FIG. 4. (Color online) Variable u of the Fitzhugh-Nagumo equation (7). (a) Free running system exhibiting chaotic dynamics. (b) Traveling waves stabilized by MDFC with three delay times $\tau_1=10.2$, $\tau_2=45$, $\tau_3=78.2$, and gains $k_{1a}=0.13$, $k_{1b}=0.31$, $k_{2a}=0.40$, $k_{2b}=0.34$, $k_{3a}=0.17$, $k_{3b}=0$ applied at the control cells indicated by white rectangles.

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